Write and equation of the line given the two points. Graph the line.

1) (1,6) (4,2)

\[
\frac{y-2}{1-6} = \frac{4}{-3} = -\frac{4}{3}
\]

\[
y - 2 = -\frac{4}{3} (x - 1)
\]

\[
y - 2 = -\frac{4}{3} + \frac{4x}{3}
\]

\[
y = -\frac{4}{3} x + 1 + \frac{4}{3} \rightarrow y = -\frac{4}{3} x + \frac{2}{3}
\]

Which sets of ordered pairs represents a function from \(x\) to \(y\).

2) a) (20,4) (40,0) (20, 6) (30, 2)
   Not \(2\) \(x\) assigned to \(2\) values

b) (10, 4) (20, 4) (30, 6) (40, 6)
   - Function

   c) (-2, 8) (-1, 1) (0, 0) (1, -1) (2, 8)
   - Function

   d) (-2, 6) (0, 6) (-2, 8) (8, 8)
   Not \(-2\) \(x\) assigned to \(2\) values

Which equation represents \(y\) as a function of \(x\). Explain.

3) a) \(16x^2 - y^2 = 0\)
   b) \(y = \sqrt{x^2 + 4}\)

Sketch the graph of each function.

4) \(w(x) = \begin{cases} 
1, & x \leq -4 \\
\frac{|x|}{2}, & -4 < x < 2 \\
-|x|, & x \geq 2 
\end{cases}\)

Using the previous piecewise function, evaluate the following.

5) a) \(w(-4)\)  
   b) \(w(-2)\)  
   c) \(w(2)\)  
   
   \[
   \left| \frac{-2}{2} \right| = 1
   \]
6) State the domain and range for each graph and determine if each graph represents a function.

7) Evaluate the following functions.

\[ h(n) = -2n^2 + 4; \text{ Find } h(4) \]
\[ h(4) = -2(4)^2 + 4 \]
\[ = -32 + 4 = -28 \]

8) Show that the following functions are inverses.

\[ f(n) = \frac{-16 + n}{4} \]
\[ g(n) = 4n + 16 \]
\[ (f \circ g)(n) = n \]
\[ (g \circ f)(n) = n \]
\[ f(4n + 16) = -16 + (4n + 16) = \frac{4n}{4} = n \checkmark \]
\[ g\left(\frac{-16 + n}{4}\right) = \frac{-16 + n}{4} + 16 = n \checkmark \]

\[ f(n) \text{ and } g(n) \text{ are inverses} \]

9) Find the inverse function of \( g(x) \)

\[ g(n) = n^3 - 5n^2; \text{ Find } g(-4n) \]
\[ g(-4n) = (-4n)^3 - 5(-4n)^2 \]
\[ = -64n^3 - 80n^2 \]

\[ \text{Switch } x \leftrightarrow y \]
\[ x = \frac{7y + 18}{2} \]
\[ 2x - 18 = 7y \]
\[ 2x - 18 = y \]
\[ g^{-1}(x) = \frac{2x - 18}{7} \]
For the following functions, identify and graph the parent function. State the transformations needed to move from the parent function to the given function, and graph the given function. Label the graphs of each function clearly.

a) \[ f(x) = (x - 2)^2 + 4 \]

Parent Function: \( g(x) = x^2 \)

b) \[ f(x) = |x - 3| - 2 \]

Parent Function: \( g(x) = |x| \)

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Find the vertex. Identify the end behavior. Plot a few points and sketch the graph.

II. \( f(x) = -x^2 + 6x - 3 \)

**Shortest Method**

\[ \begin{align*}
(x, y) & \quad \left( -\frac{b}{2a}, f\left( -\frac{b}{2a}\right) \right) \\
(1, 3) & \quad f\left( -\frac{6}{2(-1)} \right) = f(3) \\
& \quad = -(3)^2 + 6(3) - 3 \\
& \quad = -9 + 18 - 3 = 6
\end{align*} \]

\[ \text{Vertex: } (3, 6) \]

End Behavior: opens downward because \( a \) is negative

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Other points:

<table>
<thead>
<tr>
<th>[ x ]</th>
<th>[ 2 ]</th>
<th>[ 4 ]</th>
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<tr>
<td>[ f(x) ]</td>
<td>[ 5 ]</td>
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Describe the end behavior. Find x and y intercepts. Sketch the graph of each function. State the Domain and Range.

For each function: (1) state the maximum number of turns the graph could make, (2) determine the real zeros and state the multiplicity of any repeated zeros, and (3) sketch the graph.

14. \( f(x) = -x^3 + 3x^2 \)  
   - \( 0 \) Turns \( \rightarrow \) max of 2.  
   - \( \rightarrow \) Rises on \( L \), Falls on \( R \).
   - \( x = 3 \), mult. 1
   - \( x = 0 \), mult. 2

15. \( f(x) = 4x^3 + 12x^2 - 9x \)  
   - \( \rightarrow \) \( -x \) of \( 4x^2 - 12x + 9 \).  
   - \( x = 0 \), mult. 1
   - \( x = \frac{3}{2} \), mult. 2

\( f(1) = -4 + 12 - 9 = 1 \)
Simplify. Write your answer in standard form.

16. \((-3 + 2i)(-1 + 2i)\)
   \[3 - 6i - 2i + 4i^2\]
   \[3 - 8i + 4(-1)\]
   \[3 - 8i - 8\]
   \[-1 - 8i\]

17. \((3i + 7)(3 - 8i)i + 9\i\)
   \[7 - 8 + 5i + 9i\]
   \[-1 + 19i\]

Divide. Write your answer in fraction form.

20. \((x^4 + 12x^3 + 27x^2 + 19x + 11) + (x + 2)\)
   \[-2\]
   \[
   \begin{array}{rrrrr}
   -2 & 1 & 12 & 27 & 19 & 11 \\
   -2 & -20 & -14 & -10 & & \\
   1 & 10 & 7 & 5 & 11 \\
   \end{array}
   \]
   \[x^3 + 10x^2 + 7x + 5 + \frac{1}{(x+2)}\]

21. \((14 + 4x^2 + 23x) + (x + 5)\)
   \[4x^2 + 23x + 14\]
   \[-5\]
   \[
   \begin{array}{rrrrr}
   -5 & 4 & 23 & 14 \\
   -20 & -15 & & & \\
   4 & 3 & 1 & 1 \\
   \end{array}
   \]
   \[4x^2 + 3 - \frac{1}{x + 5}\]

Confirm that the function has the indicated zeros.

22. \(f(x) = x^3 + 49x; \ 0, -7i, 7i\)
   \[x (x + 7i) (x - 7i)\]
   \[x^2 - (7i)^2\]
   \[x^2 - 49(-1)\]
   \[x^2 + 49\]
Find all the zeros and write the polynomial as a product of the linear factors.

$$2g(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$$

Rational zeros: $$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$$

Using synthetic division:

$$\begin{array}{c|ccccc}
\text{Divisor} & 1 & -8 & 17 & -8 & 16 \\
\text{Quotient} & 1 & -4 & 1 & 4 & 0 \\
\end{array}$$

Linear factors:

$$f(x) = (x - 4)(x^2 + 1)$$

For each function, identify holes, x and y intercepts vertical and horizontal asymptotes, and any additional points needed. Then sketch the graph.

24. $$f(x) = \frac{-x^2 + 2}{x - 3}$$

1. No simplification
2. y-intercept: $$f(0) = -\frac{2}{3}$$
3. x-intercepts: $$-x + 2 = 0$$ 
   $$x = 2$$
4. Vertical Asymptote: $$x = 3$$
5. Horizontal Asymptote: $$y = -\frac{1}{x}$$
6. Domain: All real numbers except 3
7. Range: All real numbers

25. $$f(x) = \frac{x^2 - x}{x^2 - x - 6}$$

1. No simplification
2. y-intercept: $$f(0) = 0$$
3. x-intercepts: $$x = 0, x = 1$$
4. Vertical Asymptote: $$x = 3, x = -2$$
5. Horizontal Asymptote: $$y = \frac{x^2}{x^2} = 1$$
6. Domain: All real numbers except 2 and -3
7. Range: All real numbers except 1